

Electroproductions of Light Λ - and Σ -Hypernuclei

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Abstract

Theoretical estimations of production cross sections of light Λ and Σ hypernuclei in $(e, e'K^+)$ reactions at around CEBAF energies are given. Because of dominant spin-flip amplitudes and large momentum transfers, unnatural parity states and stretched states of hypernuclei are favorably excited. They are compared with quasifree hyperon productions.

1 Introduction

In last two decades, hypernuclear physics is remarkably developed both in theoretical and experimental aspects. The (K^-, π) and (π, K^+) experiments have been performed intensively and clarified properties of many hypernuclear states. However, such states are only a part of possible hypernuclear states. For example, ground states and deep-hole states of heavy hypernuclei are hardly excited in these experiments because of strong absorption of π and K^- in nuclear medium. Unnatural parity states are also weakly excited because of the small spin-flip amplitudes.

As alternative tools to produce hypernuclei, photo- and electro-productions of hypernuclei, that is, hypernuclear productions in the (γ, K^+) and $(e, e'K^+)$ reactions, are discussed. These reactions contain only particles which interact weakly with nuclear medium. Therefore, these are favorable to produce directly hypernuclei with a deeply bound Λ and/or a nucleon deep-hole. Further, the dominant spin-flip amplitudes in these reactions are of great advantage to excite unnatural parity states of hypernuclei.

Theoretical estimations[1]-[8] of photoproductions of hypernuclei have been performed by several authors. But the electroproductions[1] were treated under rough approximations or only for the Λ -hypernuclei[9]. In the present work, to improve such a situation, we give theoretical estimations of the electroproduction cross sections both for Λ and Σ -hypernuclei. In spite of uncertainties in

the elementary reactions, we treat the energy region corresponding to CEBAF, which gives high possibility to the experiment in near future.

Our calculations are based on a relativistic impulse approximation and the Walecka model of nuclei and hypernuclei. The elementary reactions are treated relativistically without any approximation within the model. Parameters in the model are determined to reproduce known experimental data.

In §2, we give the model of elementary processes. In §3, the relativistic model for nuclei and hypernuclei are explained. In §4, the relativistic impulse approximation for hypernuclear production is formulated. Numerical results are given in §5. The results are compared with the quasifree hyperon production which is the largest background process in the experiments in §6. Conclusions of this work is given in §7.

2 Elementary processes $N(e, e'K^+)Y (Y = \Lambda, \Sigma)$

Theoretical models of processes including the strong interaction cannot be determined uniquely. We can only pick up possible mechanisms and determine the parameters in the model so as to reproduce known empirical data. We assume that the processes $N(e, e'K^+)Y$ consist of one-photon-exchange and $N(\gamma, K^+)Y$ vertex, as shown Fig.1a. The former is theoretically undoubted because of the small QED coupling constant. The latter is complicated and is understood only insufficiently.

As a model of the $N(\gamma, K^+)Y$ vertex, we employ a sum of one-particle-exchange mechanisms as Fig.1b. This model is commonly used in theoretical studies[10]-[14]. The coupling constants in the model are determined by fitting the experimental data. In the present work, we use the values in refs.[1] and [6], which are listed in Table 1 and 2 for $Y = \Lambda$ and Σ , respectively. It should be noted that the model and the values of parameters have large uncertainties because of the limitation of experimental data.

Details of calculations with these parameters are given in refs.[1] and [6]. To check our computer code, we repeated their calculations. As a result, we obtained almost the same results with those in the original works. Only one exception is an oppsite assignment of lines in the Fig.2 of ref.[1].

Using the model, we calculate the elementary processes $N(e, e'K^+)Y$ at around CEBAF energies. As expected, we find vary large spin-flip amplitudes in all cases.

As a comparison, we can also use the approximate expression,

$$\frac{d^3\sigma}{d\Omega_{e'}dE_{e'}d\Omega_K} = \frac{1}{2\pi^2} \frac{\alpha}{1-\epsilon} \frac{E_{e'}}{E_e} \frac{k_\gamma}{(E_\gamma^2 - k_\gamma^2)} \frac{d\sigma}{d\Omega_K}(\gamma, K), \quad (1)$$

which is used in ref.[1]. In this expression, only on-shell $N(\gamma, K^+)Y$ amplitudes are assumed. But this gives different results from our exact calculations by around 20-40%.

Table 1: Coupling constants^{a)} for $\gamma + p \rightarrow K^+ + \Lambda$.

$\frac{g_\Lambda}{\sqrt{4\pi}}$	$\frac{G_\Sigma}{\sqrt{4\pi}}$	$\frac{G_V}{4\pi}$	$\frac{G_T}{4\pi}$
2.57	1.52	0.105	0.064

a)These are assigned by set1 in ref.[1].

Table 2: Coupling constants^{a)} for $\gamma + N \rightarrow K^+ + \Sigma$.

	$\frac{g_\Sigma}{\sqrt{4\pi}}$	$\frac{G_\Lambda}{\sqrt{4\pi}}$	$\frac{G_V}{4\pi}$	$\frac{G_T}{4\pi}$	$\frac{G_\Delta^1}{4\pi}$	$\frac{G_\Delta^2}{4\pi}$
PS	2.20	-4.82	0.113	-0.038	0	0
PS+delta	2.72	-3.60	0.104	0.005	-0.069	0.314

a)These are taken from ref.[6]

It should be noted that the present model give large cross sections for $n(e, e'K^+)\Sigma^-$ in comparison with those for $p(e, e'K^+)\Sigma^0$ by about one order of magnitude. Therefore, the Σ hypernuclear productions are sensitive to neutron wave functions in the target nucleus

3 Relativistic model of nuclei and hypernuclei

In the present work, we treat relativistically elementary reactions and hypernuclear productions. For consistency, we should employ a relativistic model of nuclei and hypernuclei. The most famous is the Walecka model. In this model, nucleons and hyperons are assumed to behave as Dirac particles and are moving

Table 3: Potentials for nucleon and calculated single particle energies for protons(in MeV and fm)

	⁴ He	¹² C	¹⁶ O
S_0	-286.0	-288.0	-426.0
V_0	200.0	215.0	340.0
a	-	0.7	0.7
r_0	1.2	1.2	1.055
$s_{1/2}$	-20.1	-34.9	-40.3
$p_{3/2}$	-	-15.3	-18.3
$p_{1/2}$	-	-	-12.0

Table 4: Potentials for hyperons and calculated single particle energies(in MeV and fm)

	${}^4_{\Lambda}\text{H}$	${}^4_{\Lambda}\text{H}^*$	${}^{16}_{\Lambda}\text{N}$	${}^{12}_{\Lambda}\text{B}$	${}^4_{\Sigma}\text{H}({}^4_{\Sigma}\text{H}^*)$	${}^{16}_{\Sigma}\text{N}$	${}^{12}_{\Sigma}\text{B}$
S_0	-142.5	-137.2	-187.0		-243.0	-187.0	
V_0	100.0	100.0	155.0		200.0	177.0	
a	-	-	0.7			0.7	
r_0	1.0	1.0	1.15		1.0	1.15	
$s_{1/2}$	-2.1	-1.0	-13.3 ^{a)}	-11.6	-3.0	-5.1	-3.5
$p_{3/2}$	-	-	-2.7 ^{a)}	-1.1	-	-	-
$p_{1/2}$	-	-	-1.7 ^{a)}	-0.3	-	-	-
a)for ${}^{15}_{\Lambda}\text{N}$ (see text)							

in attractive scalar potential and repulsive vector potential in nuclear medium. The Dirac equation for nucleon(hyperon) is given by

$$[i\vec{\nabla} - m - S(r) - \gamma_0 V(r)]\psi(\mathbf{r})e^{-iEt} = 0, \quad (2)$$

where, m is the reduced mass between nucleon(hyperon) and core nucleus. $S(r)$ and $V(r)$ are the scalar and vector potentials for nucleon (hyperon) in nuclei (hypernuclei). This equation is solved numerically and the single particle energy is given by $E - m$.

The potentials $S(r)$ and $V(r)$ are determined so as to reproduce phenomenological single particle energies(SPE) in nuclei and hypernuclei. In this work, except for the $A = 4$ cases, the potentials are assumed to have the Woods-Saxon shape like as

$$S(r) = S_0[1 + \exp((r - R)/a)]^{-1}, \quad (3)$$

$$V(r) = V_0[1 + \exp((r - R)/a)]^{-1}. \quad (4)$$

For $A = 4$ cases, the Gaussian shape is assumed;

$$S(r) = S_0 \exp(-(r/R)^2), \quad (5)$$

$$V(r) = V_0 \exp(-(r/R)^2). \quad (6)$$

$S_0, V_0, R = A^{1/3}r_0$, and a used in the present calculations are displayed in Table 3 and 4.

The potentials for nucleons are given in Table 3. In ${}^{16}\text{O}({}^{16}_{\Lambda}\text{N})$ and ${}^{12}\text{C}({}^{12}_{\Lambda}\text{B})$, the potentials are determined to give phenomenological proton SPE in $s_{1/2}, p_{3/2}$ and $p_{1/2}$ (only for ${}^{16}\text{O}({}^{16}_{\Lambda}\text{N})$) states. On the other hand, those in ${}^4\text{He}({}^4_{\Lambda}\text{H})$ are determined so as to reproduce the binding energy and the root-mean-square radius of ${}^4\text{He}$, simultaneously.

For hyperons, the potentials are given in Table 4. Those for the Λ particle give phenomenological SPE in $^{15}_{\Lambda}\text{N}$ (those in $^{16}_{\Lambda}\text{N}$ are not available) and $^{12}_{\Lambda}\text{B}$. We can easily understand weaker strengths of S and V than those for nucleon by the small spin-orbit splitting for the Λ particle. The potentials in $^4_{\Lambda}\text{H}$ and $^4_{\Lambda}\text{H}^*$ are determined to reproduce their experimental Λ separation energies, respectively.

For Σ particle, phenomenological informations are very limited. Therefore, we assume some trial values. In $^4_{\Sigma}\text{H}$, the potential for Σ are determined to give $B_{\Sigma} = 3.0\text{MeV}$, which is near the experimental data[15] and theoretically predicted values[16]-[18]. $^4_{\Sigma}\text{H}^*$ with spin=1 is a hypothetical object with the same binding energy with $^4_{\Sigma}\text{H}$. This is introduced in order to clarify features of the electroproductions. In $^{16}_{\Sigma}\text{N}$ and $^{12}_{\Sigma}\text{B}$, the potentials give -5.1MeV and -3.5MeV for $s_{1/2}$ SPE, respectively. In both systems, all p -states are unbound.

It should be noted that the quantitative modifications of the potentials are not so sensitive to qualitative features of hypernuclear production discussed later, because they are mainly determined by the angular momentum and parity selections and high momentum components of wave functions, which is weakly influenced by the binding energies.

4 Hypernuclear productions in Relativistic Impulse Approximation

Since both the incident and outgoing particles have high energies compared with nucleon binding energies in nuclei and momentum transfers in the reaction are considerably larger than Fermi momenta of nuclei, the relativistic impulse approximation is expected to work well. In fact, this approximation is used successfully in various hypernuclear production reactions.

In the relativistic impulse approximation, the hypernuclear production cross section in the $(e, e'K^+)$ reaction is given by

$$d\sigma = (2\pi)^4 \delta^4(p_e + p_A - p_{e'} - p_B - p_K) \frac{d^3\mathbf{p}_{e'} m_e}{(2\pi)^3 E_{e'}} \frac{d^3\mathbf{p}_K}{(2\pi)^3 E_K} \frac{d^3\mathbf{p}_B}{(2\pi)^3} \quad (7)$$

$$\frac{(2m_p)(2m_e)}{[(p_e \cdot p_A)^2 - p_e^2 p_A^2]^{1/2}} \frac{1}{2J_i + 1} \sum_{M_i, M_f} |\langle J_f M_f T_f N_f | T | J_i M_i T_i N_i \rangle|^2,$$

where, $p_A(p_B)$ and $|J_i M_i T_i N_i\rangle$ ($|J_f M_f T_f N_f\rangle$) are the momentum and the quantum state of the initial nucleus (the final hypernucleus), respectively. The T -matrix is given by the nuclear matrix elements and the elementary process as follows;

$$\langle J_f M_f T_f N_f | T | J_i M_i T_i N_i \rangle = \sum_{\alpha\alpha'} \langle J_f M_f T_f N_f | C_{\alpha'}^\dagger C_{\alpha} | J_i M_i T_i N_i \rangle \langle \alpha' | t | \alpha \rangle, \quad (8)$$

where, α is a set of single particle quantum numbers, $\{n, j, m; t, m_t\}$. In our simple model without configuration mixings, the nuclear matrix element given by

$$\begin{aligned} \langle J_f M_f T_f N_f | C_{\alpha'}^\dagger C_\alpha | J_i M_i T_i N_i \rangle &= \sum_{JMTN} (-1)^{j-m+1/2-m_t+J_i-M_i+T_i-N_i} \\ &\hat{j}^2 \hat{T}^2 \begin{bmatrix} J_f & J_i & J \\ -M_f & M_i & M \end{bmatrix} \begin{bmatrix} T_f & T_i & T \\ -N_f & N_i & N \end{bmatrix} \begin{bmatrix} j' & j & J \\ -m' & m & M \end{bmatrix} \begin{bmatrix} t_y & 1/2 & T \\ -m_y & m_t & N \end{bmatrix}, \quad (9) \end{aligned}$$

for a definitely occupied nucleon state ($\alpha = \{n, j, m, 1/2, m_t\}$) in the initial nuclear state and a hyperon state ($\alpha' = \{n', j', m', t_y, m_y\}$) in the final hypernuclear state.

The elementary process in nuclear medium is given by

$$\langle \alpha' | t | \alpha \rangle = \int d^3 \mathbf{p} d^3 \mathbf{q}' \bar{\psi}_{\alpha'}(\mathbf{p}') \phi_{K^+}^{(-)*}(\mathbf{p}_K, \mathbf{q}') t(p_e, p; p_{e'}, p', q') \psi_\alpha(\mathbf{p}). \quad (10)$$

In the present work, we use the plane-wave ($\delta^3(\mathbf{p}_K - \mathbf{q}')$) instead of the distorted wave ($\phi_{K^+}^{(-)*}(\mathbf{p}_K, \mathbf{q}')$) for outgoing kaon, because kaons with energies near 1 GeV does not strongly interact with nuclei and because only light nuclear targets are considered. The effects of the distortion of kaons by nuclear interaction were discussed in the photoproductions by several authors[4][8]. The authors showed that the effects reduce the cross sections by about 20-30% for ^{16}N . We can expect the effects are less important for lighter hypernuclei.

5 Numerical Results

A large advantage of $(e, e' K^+)$ is a variety of the kinematical condition. We can select the four momentum of the intermediate photon through the final electron(e') energy and angle. To avoid large momentum transfers, which prevent hyperons from sticking on nuclei, we should select suitable kinematical conditions.

As an example, we show the kinematics for $^{16}\text{O}(e, e' K^+)^{16}_\Lambda\text{N}$ in Fig.2. Fig.2a shows that if final electrons(e') are detected at a finite angle($\theta_{e'} > 0^\circ, \phi_{e'} = 0^\circ$), the angle for outgoing K minimizing the momentum transfer(p_B) is given by $\theta_K \sim \theta_{e'}, \phi_K = 180^\circ$. Further, from Fig.2b we find that in the finite-angle case ($\theta_K \sim \theta_{e'} > 0$) as mentioned above, there is a final electron energy minimizing p_B , which is around 1.2GeV in the case of Fig.2b. Our calculations are performed near such suitable kinematical conditions.

To examine what kinds of hypernuclear states are favorably excited in the reactions, we calculate the production cross sections for various hypernuclear states under a common kinematical condition, that is,

$$p_e = 3.0 \text{ GeV}/c,$$

$$\begin{aligned}
p_{e'} &= 1.2\text{GeV}/c, \\
\theta_{e'} &= 6^\circ, \phi_{e'} = 0^\circ, \\
\theta_K &= 10^\circ, \phi_K = 180^\circ,
\end{aligned} \tag{11}$$

in the laboratory frame.

The results are given in Tables 5-9. From these tables, we apparently find that the states with the largest J_f for given nucleon-hole and hyperon states, that is, the stretched states are strongly excited. This can be easily understood by large momentum transfers in the reactions. However, the same reason makes absolute cross sections small. For $^{16}_\Lambda\text{N}$ and $^{12}_\Lambda\text{B}$ cases, the $[(p_{3/2})_p^{-1}, (p_{3/2})_\Lambda]_{J_f=3}$ states are most favorably excited with cross sections 2.485 and 2.667 nb/sr²/GeV, respectively. These values correspond to about 1.3% in the sticking probability, which is the ratio to $(2j+1) \times$ (cross section of the elementary process). For $^4_\Lambda\text{H}^*$, the cross section (2.260nb/sr²/GeV) is much larger than those for the $[(s_{1/2})_p^{-1}, (s_{1/2})_\Lambda]_{J_f=1}$ states in $^{16}_\Lambda\text{N}$ and $^{12}_\Lambda\text{B}$. The reason is that recoils of light hypernuclei reduce effectively the momentum transfers in the hypernuclear center-of-mass frame (by $(A-1)/A$).

Similar behaviors can be seen for Σ hypernuclear productions. Σ hypernuclear states with $T_f = 3/2$ can be excited. But, their cross sections are much smaller than those with $T_f = 1/2$ by about two orders of magnitude. Therefore, we show results only for $T_f = 1/2$.

Another prominent feature is large cross sections for unnatural parity states. This is due to the dominant spin-flip amplitudes in the elementary processes $N(\gamma, K^+)Y$. In fact, cross sections for $^4_\Lambda\text{H}^*$ and $^4_\Sigma\text{H}^*$ are larger than those for $^4_\Lambda\text{H}$ and $^4_\Sigma\text{H}$ by two orders of magnitude, respectively. At more forward angles ($\theta_K < 10^\circ$), these ratios become larger, as shown Fig.5. Such a behavior is one of typical characters of spin-flip processes. Small cross sections for $[(p_{3/2})_p^{-1}, (p_{3/2})_\Lambda]_{J_f=2}$ states of $^{16}_\Lambda\text{N}$ and $^{12}_\Lambda\text{B}$ are also an evidence of the spin-flip nature.

In Tables 7-9, we give results of Σ hypernuclear productions for two kinds of the elementary process, that is, PS and PS+delta models. Bennhold[6] noted that PS+delta model give a little better χ^2 than PS model for known data but the final conclusion would have to await further data. The differences between them, which are 30%-50%, may be regarded as a scale of uncertainties.

Angular distributions are shown in Figs.3-5. We can see that except for $A=4$ cases (Fig.5), the cross sections reduce rapidly for larger angles than 10° . In the present kinematics, $\theta_K \sim 4^\circ$ gives the minimum momentum transfer $\sim 270\text{MeV}/c$ and $\theta_K = 10^\circ(15^\circ)$ does $\sim 320(420)\text{MeV}/c$. In other word, for $\theta_K > 10^\circ$, momentum transfers go rapidly away from the Fermi momentum ($\sim 280\text{MeV}/c$).

For $A=4$ cases, a factor $(A-1)/A$ mentioned above makes the reduction mild, as shown in Fig.5. A peculiar behavior for $^4\text{H}(e, e'K^+)^4_\Lambda\text{H}(^4_\Sigma\text{H})$ comes from the dominant spin-flip amplitudes in the elementary reactions.

Table 5: Production cross sections for various states of $^{16}_{\Lambda}\text{N}$ in nb/sr²/GeV

hole	Λ	J_f		hole	Λ	J_f	
$s_{1/2}$	$s_{1/2}$	0	0.002	$p_{3/2}$	$p_{3/2}$	2	0.013
		1	0.393			3	2.485
	$p_{3/2}$	1	0.203		$p_{1/2}$	1	0.021
		2	0.631			2	1.329
	$p_{1/2}$	0	0.002	$p_{1/2}$	$s_{1/2}$	0	0.033
		1	0.369			1	0.844
$p_{3/2}$	$s_{1/2}$	1	0.526	$p_{3/2}$		1	0.023
		2	1.576			2	1.320
	$p_{3/2}$	0	0.000	$p_{1/2}$		0	0.002
		1	0.388			1	0.555

Table 6: Production cross sections for various states of $^{12}_{\Lambda}\text{B}$ in nb/sr²/GeV

hole	Λ	J_f		hole	Λ	J_f	
$s_{1/2}$	$s_{1/2}$	0	0.003	$p_{3/2}$	$p_{3/2}$	0	0.000
		1	0.644			1	0.390
	$p_{3/2}$	1	0.223			2	0.020
		2	0.709			3	2.667
	$p_{1/2}$	0	0.003	$p_{1/2}$		1	0.024
		1	0.354			2	1.252
$p_{3/2}$	$s_{1/2}$	1	0.724				
		2	2.185				

Table 7: Production cross sections for various states of $^{16}_{\Sigma}\text{N}(T_f = 1/2)$ in nb/sr²/GeV

hole	Σ	J_f	PS+delta	PS
$s_{1/2}$	$s_{1/2}$	0	0.001	0.001
		1	0.013	0.005
$p_{3/2}$		1	0.077	0.055
		2	0.314	0.176
$p_{1/2}$		0	0.055	0.021
		1	0.093	0.056

Table 8: Production cross sections for various states of $^{12}_\Sigma\text{B}(T_f = 1/2)$ in nb/sr²/GeV

hole	Σ	J_f	PS+delta	PS
$s_{1/2}$	$s_{1/2}$	0	0.001	0.001
		1	0.029	0.018
$p_{3/2}$		1	0.112	0.085
		2	0.420	0.276

Table 9: Production cross sections for ^4_YH and $^4_Y\text{H}^*$ in nb/sr²/GeV

J_f			J_f		
				PS+delta	PS
$^4_\Lambda\text{H}$	0	0.020	$^4_\Sigma\text{H}$	0	0.054
$^4_\Lambda\text{H}^*$	1	2.260	$^4_\Sigma\text{H}^*$	1	4.021
					2.600

6 Comparison with Quasifree Hyperon Production

In the experiments, quasifree hyperon productions are the largest background process for hypernuclear productions. To examine the feasibility of the experiments, we have to estimate this background process.

Quasifree hyperon production cross sections are given by

$$\frac{d\sigma}{d\omega} = \alpha\sigma R(q, \omega), \quad (12)$$

where σ is the free elementary cross section, $R(q, \omega)$ is so-called a quasifree response function and α is a kinematical factor near 1.

Based on the picture of knockout processes from light nucleus, $R(q, \omega)$ is given by

$$R(q, \omega) = (2\pi) \int_S dp_N \frac{p_N}{q} \rho(p_N) |\omega + M_i - (M_f^2 + p_N^2)^{1/2}|, \quad (13)$$

where, $M_i(M_f)$ is rest mass of initial target(final residual) nucleus and $\rho(p_N)$ is the nucleon momentum distribution in target nucleus. The region S is defined by

$$S = \left\{ p_N \left| p_N \geq 0, \sqrt{m_Y^2 + (p_N - q)^2} \leq \omega + M_i - \sqrt{M_f^2 + p_N^2} \leq \sqrt{m_Y^2 + (p_N + q)^2} \right. \right\}.$$

Using this formalism, we obtain the result shown in Fig.6 for the case of ${}^4\text{He}$ target. The result depends strongly on the widths of hypernuclei. If the hypothetical ${}^4_\Sigma\text{H}^*$ exists, we can observe clearly its signal. However, the signals of more possible states, for example, $[(p_{3/2})_N^{-1}, (s_{1/2})_\Sigma]_{J_f=2}$ states in ${}^{16}_\Sigma\text{N}$ and ${}^{12}_\Sigma\text{B}$ may be not so clear, because of their small cross sections (0.3-0.4 nb/sr²/GeV), which correspond to about 10% of that for ${}^4_\Sigma\text{H}^*$ (see filled peaks in Fig.6).

For Λ hypernuclei, Fig.6 shows that the electroproduction experiment is promising. We can expect to observe two or three unnatural parity states for in ${}^{16}_\Lambda\text{N}$ and ${}^{12}_\Lambda\text{B}$.

7 Conclusion

In the present work, we estimated the electroproduction of hypernuclei. We find large cross sections for the stretched states and unnatural parity states. For Λ hypernuclei, they are a few nb/sr²/GeV, which seems sufficiently measurable in spite of large backgrounds by the quasifree hyperon productions. For Σ hypernuclei, we find the cross sections of several tenth of nb/sr²/GeV for ${}^{16}_\Sigma\text{N}$ and ${}^{12}_\Sigma\text{B}$. These signals may not be measured clearly in the experiments. In these calculations, however, all p -states of Σ particle were assumed to be unbound. If the bound p -states exist, we get measurable cross sections (a few nb) for the stretched states including such a p -state Σ . If ${}^4_\Sigma\text{H}^*$ with spin=1, which is not bound theoretically [16]-[18], exists, it can be clearly observed in the electroproduction experiments.

The present calculations include several uncertainties. Especially, the elementary processes at the energy region considered here is little known. Their phenomenological data are strongly desired.

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Figure captions.

Fig.1 a: Model of $N(e, e' K^+)Y$ reactions. b: Model of $N(\gamma, K^+)Y$ vertex. ($Y = \Lambda, \Sigma$)

Fig.2 Momentum transfer to hypernuclei as a function of K^+ angle, θ_K (a) and final electron momentum, $p_{e'}$ (b) in the case of $^{16}\text{O}(e, e' K^+)_{\Lambda}^{16}\text{N}(p_e = 3.0\text{GeV}/c, \theta_{e'} = 10^\circ, \phi_{e'} = 0^\circ)$. a: Solid and dotted lines are the cases with $\phi_K = 0^\circ$ and 180° , respectively ($p_{e'} = 1.2\text{GeV}/c$). b: Solid and dotted lines are the cases with $\theta_K = 0^\circ$ and 10° , respectively ($\phi_K = 180^\circ$).

Fig.3 Angular distributions of productions of Λ hypernuclear stretched states in $^{16}\text{O}(e, e'K^+)_{\Lambda}^{16}\text{N}$ (a,b) and $^{12}\text{C}(e, e'K^+)_{\Lambda}^{12}\text{B}$ (c). Solid, dotted and dashed lines are those for hypernuclear states with the $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$ Λ states, respectively. Lines in a(upper and lower lines in b for each type of line) are those with the $s_{1/2}(p_{3/2}$ and $p_{1/2})$ nucleon hole. Lower(upper) three lines in c are those with the $s_{1/2}(p_{3/2})$ nucleon hole. $p_e=3.0\text{GeV}/c$, $p_{e'}=1.2\text{GeV}/c$, $\theta_{e'} = 6^\circ$, $\phi_{e'} = 0^\circ$, $\phi_K = 180^\circ$.

Fig.4 The same as Fig.3 but in $^{16}\text{O}(e, e'K^+)_{\Sigma}^{16}\text{N}$ and $^{12}\text{C}(e, e'K^+)_{\Sigma}^{12}\text{B}$. In all cases, Σ states are $s_{1/2}$. Solid, dotted, dashed and dash-dotted lines are those with the $s_{1/2}(J_f = 1)$, $p_{3/2}(J_f = 1)$, $p_{3/2}(J_f = 2)$ and $p_{1/2}(J_f = 1)$ nucleon hole, respectively.

Fig.5 The same as Fig.3 but in $^4\text{He}(e, e'K^+)_{\Lambda}^4\text{H}$, $_{\Lambda}^4\text{H}^*$, $_{\Sigma}^4\text{H}$ and $_{\Sigma}^4\text{H}^*$, which correspond to lower solid, upper solid, lower dotted and upper dotted lines, respectively.

Fig.6 Sum of quasifree hyperon productions and hypernuclear productions in $^4\text{He}(e, e'K^+)X$ at $\theta_K = 10^\circ$ (a) and 5° (b)($\phi_K = 180^\circ$). Peaks labeled Λ and Σ are $_{\Lambda}^4\text{H}^*(\Gamma=4\text{MeV})$ and $_{\Sigma}^4\text{H}^*(\Gamma=5\text{MeV}$ and $10\text{MeV})$ productions, respectively. Filled peaks correspond to 10% of $_{\Sigma}^4\text{H}^*(\Gamma=5\text{MeV})$ peaks(shifted to the $B_{\Sigma}=5.1\text{MeV}$ for $_{\Sigma}^{16}\text{N}$).

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